

GWS 3-30

(a) By symmetry, P_x, P_y vanish.

$$P_z = \int z \hat{z} \sigma \, da = \int R \cos \theta \, k \cos \theta \hat{z} R^2 \sin \theta \, d\theta \, d\phi$$

$$= \int R^3 k \cos^2 \theta \hat{z} \sin \theta \, d\theta \, d\phi$$

$$= k R^3 2\pi \hat{z} \int_0^\pi \cos^2 \theta \sin \theta \, d\theta$$

let $x = \cos \theta$ $d\theta = \frac{1}{-\sin \theta} dx$

$$= k R^3 2\pi \hat{z} \int_{-1}^1 x^2 dx$$

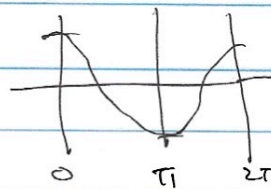
$$= k R^3 2\pi \hat{z} \int_{-1}^1 x^2 dx$$

$$= k R^3 2\pi \hat{z} \left. \frac{1}{3} x^3 \right|_{-1}^1$$

$$= \boxed{\frac{4\pi k R^3}{3} \hat{z}}$$

(b) $\cos \theta$ on $0, \pi$:

\Rightarrow monopole vanish.



$$\Phi = k_0 \frac{\vec{P} \cdot \hat{r}}{r^2} = k_0 \frac{4\pi k R^3}{3} \frac{\hat{z} \cdot \hat{r}}{r^2} = \boxed{\frac{k_0 4\pi R^3}{3} \frac{\cos \theta}{r^2}}$$

$$\frac{1}{\epsilon_0} \frac{k R^3}{3 r^2} \cos \theta = \boxed{\frac{k R^3}{3 \epsilon_0 r^2} \cos \theta}$$